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The diurnal variation of global radiation on a horizontal surface - with special reference to Northern Scotland

by R W GLOYNE

INTRODUCTION

Information is required by a range of interests on the variation during the day of the intensity of global radiation (and of illumination) on a horizontal surface. Some inquirers require estimates of the length of period when the intensity was above or below or between stated levels.

Centres providing direct data on an hourly basis are relatively few, although there are more which record the daily integral - furthermore, much of the readily available published data is in the form of daily integrals, meaned over varying periods and generally for calendar months. In addition one can - with varying degrees of accuracy - estimate the mean daily integral from data on the duration of bright sunshine.

The present contribution draws attention to certain useful empirical methods of representing the diurnal variation of mean daily global radiation, and tests the accuracy of some derived estimates against twelve months of data obtained from a recently installed solarimeter at the University of Aberdeen. It was the lack of data from regions such as north-east Scotland and the hazards of interpolating from, for example, data from long period stations such as the Observatories at Lerwick and Eskdalemuir, which led to the case-study from which this paper has been compiled.

In passing it should be noted that it is not necessarily useful if acceptable statistical relationships are obtainable only for mean values over long periods; in many practical cases relationships are needed which can be used for periods of individual months or even shorter duration.

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ANALYSIS

For a given latitude and solar declination, global radiation on a horizontal surface is broadly a function of solar altitude which in turn is a function of $\cos(H)$ where H is the local hour angle.

Trial plots of the diurnal march of global radiation, when meaned over a sufficient number of days (say 7 to 10 at least, and more generally for a month), suggest that the course is approximately sinusoidal. Monteith (1965) has stated that there is a useful relationship with that portion of a sine wave comprised within the argument $\pm \pi/2$ from the maximum point ie that at local apparent noon. The writer, whose immediate interest was in the circumstances at the ends of the day ie at times of relatively low intensities, suggests that the relevant portions of the diurnal curve may possibly be better reproduced by the "tails" of a complete sine wave.

(i) Monteith's method

In Fig 1(a) let:

$G(t)$ = intensity of global radiation on a horizontal surface at time t (measured from sunrise),

$G(x)$ = that at local apparent noon, and assumed the maximum for the day,

T = day length,

S = daily integral and equal to the area under the curve,

$\delta = \pi t/T \quad (0 \leq t \leq T)$

then with origin at sunrise we have:

$$G(t) = G(x) \sin \delta \quad \text{_____} (1)$$

It is easy to show that

$$S = G(x) \cdot 2T/\pi$$

and hence

$$G(t) = \left(\frac{\pi S}{2T} \right) \sin \pi t/T$$

Also for a given threshold intensity G we have

$$r = (T/\pi) \sin^{-1} [G/G(x)]$$

where $2r$ = total period when $G(t) < G$

and $T - 2r$ = the duration of global radiation of an intensity above the threshold values.

(ii) Gloyne's method

Using dashed symbols $G^1(t)$ G^1 $G^1(x)$ and S and $\theta = 2\pi t/T$ ($0 \leq t \leq T$)

the alternative expressions are (Fig 1b)

$$G^1(t) = \frac{1}{2} G^1(x) \cdot (1 - \cos \theta) \quad (3)$$

$$S = \frac{1}{2} G^1(x) \cdot T$$

$$\therefore G^1(t) = S/T (1 - \cos \theta) \quad (4)$$

$$1 - G^1(t) T/S = \cos \theta = \cos 2t/T \cdot \pi$$

and for a given threshold G we have

$$r = (T/2\pi) \cos^{-1}(1 - G/S \cdot T)$$

when $2r$ is the total period when $G(t) < G$ and $T - 2r$ is the period with global radiation above the threshold intensity.

Since $\theta = 2\delta$ Eq (4) may be written

$$G^1(t) = S/T \cdot 2 \sin^2 \delta \quad (5)$$

A preliminary comparison of estimates is informative. Consider the value of $G(t)/(S/T)$ at various values of t

$t =$	0	$T/6$	$T/4$	$T/3$	$T/2$
Monteith (1)	0	.79	1.11	1.36	1.57
Gloyne (2)	0	.50	1.00	1.50	2.00

The difference in the shape of the curves is evident. From $t = 0$ to beyond $t/4$ Method (i) gives the higher value; equality is reached at $\sin^{-1}\pi/4$ (about $51\frac{1}{2}^\circ$ or $T/3.5$) after which Method (ii) gives the higher estimate and an indicated maxima at local noon approaching 1.3 times that for Method (i). Of necessity the bounding curve for Method (ii) is narrower than for Method (i). The important point is, of course, which of the two approximates best to the observed values.

(iii) For future reference, we may note an expression relating to the arithmetic mean of the two estimates, viz

$$\frac{1}{2}[G(t) + G^1(t)] = S/2T \cdot \sin \delta (\pi/2 + 2 \sin \delta) \quad (5)$$

(iv) In Fig 2 the basic features of the results are illustrated using data for a month (July 1970) not included in the original data.

RESULTS

1. Hour by hour actual values and estimates by both methods of the mean intensities for each month in the period June 1966 to May 1967 (inclusive) were plotted on convenient scale (specifically: abscissa $\frac{1}{2}$ in. = 1 hour, and ordinate 1 in. = 10 mW/cm² for all except winter months of November to February when a large scale (usually 4 ins. = 10 mW/cm² was used).

A check on the accuracy of free-hand drawing of the diurnal curves was made by measuring the area under the curves and comparing the results with mean daily integral as recorded: the errors were less than $\pm 1\%$ for all months except Oct (+1.2%) and January (+2.8%).

A particular interest lay in the illumination intensity above 800 ft candles: this was taken as equivalent to 7 mW/cm² (approx) global radiation. Differences in the length of the intercepts at this level, as given by the two methods, were measured off the curves for each month; in addition the errors involved in using the mean value of the two estimates were noted.

TABLE 1 DIFFERENCE BETWEEN ESTIMATED AND ACTUAL DURATIONS ABOVE 7 mW/cm² (MEAN MONTHLY BASIS) (IN HOURS AND TENTHS)

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Method (1) - observed	0.3	0.5	0.7	0.7	1.2	2.0	1.7	1.0	0.9	0.5	0.3	0.0
Method (2) - observed	0.2	-0.7	-0.8	-1.1	-0.9	-0.1	-0.6	-1.0	-0.7	-0.4	0.0	0.9
Average of [(1) + (2)] - observed	0.3	-0.1	-0.1	-0.2	0.1	0.9	0.6	0.0	0.1	0.0	0.1	0.5

Predictably, at a level of intensity, which - except in the winter months - cuts the curve towards its base, Method (1) gives an overestimate and Method (2) an underestimate of the duration; Method (2), in this particular sample, is marginally better than Method (1). It will be noted that a straight arithmetic mean gives good agreement (except perhaps in June and in December when actual intensity did not rise above 7 mW/cm²).

Since we are concerned with short-periods, 10-day means were formed (1-10, 11-20, 21-30, 31 or 28), the actual data and estimates from Method (2) plotted: the intercept at 7 mW/cm² were measured from the "actual" and the estimated curves. The errors incurred were almost all underestimates with an absolute value of one hour or less. Accordingly from this particular sample we might conclude that the methods are capable of giving results for mean over 10-days as reliably as over a calendar month.

2. The results illustrated in Fig 2 merit some attention. The intensities at (hh + 30) as observed, and as estimated, are given in Table 2.

Table 2. Intensity (in mW/cm^2) of global radiation throughout the day (mean values for Aberdeen, July 1970) as observed and as computed; together with errors of estimate

- (a) Actual values
 (b) according to Monteith - Method (1)
 (c) according to Gloyne - Method (2)
 (d) by the mean of (b) and (c)
 (e) by displacing curve (c) by 30 mins viz associating (a) for (hh + 30) with (c) for (hh + 60)

Time Method	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19
	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30
(a)	2.4	7.4	14.9	23.0	28.2	32.8	39.8	36.8	39.0	33.5	26.9	21.6	16.7	10.7	5.5	1.8
(b)	6.4	11.8	16.8	21.3	25.0	27.9	29.9	31.0	31.0	29.9	27.9	25.0	21.3	16.8	11.8	6.4
(b) - (a)	4.0	4.4	1.9	-1.7	-3.2	-4.9	-9.9	-5.8	-8.0	-3.6	1.0	3.6	4.6	6.1	6.3	4.6
(c)	1.7	5.7	11.6	18.6	25.5	31.9	36.7	39.3	39.3	36.7	31.9	25.5	18.6	11.6	5.7	1.7
(c) - (a)	-0.7	-1.7	-3.2	-4.4	-2.7	-0.9	-3.1	+2.5	+0.3	+3.2	+5.0	+3.9	+1.9	+0.9	+0.2	-0.1
(d)	4.1	8.7	14.2	20.0	25.3	29.9	33.3	35.1	35.1	33.3	29.9	25.3	20.0	14.2	8.7	4.1
(d) - (a)	1.7	1.3	-0.7	-3.0	-2.9	-2.9	-6.5	-1.7	-3.9	-0.2	3.0	3.7	3.3	3.5	3.2	2.3
(e)	3.4	8.5	15.0	22.1	28.9	34.5	38.3	39.6	38.3	34.5	28.9	22.1	15.0	8.5	3.4	0.5
(e) - (a)	1.0	1.1	0.1	-0.9	0.7	1.7	-1.5	2.8	-0.7	1.0	2.0	0.5	-1.7	-2.2	-2.1	-1.3

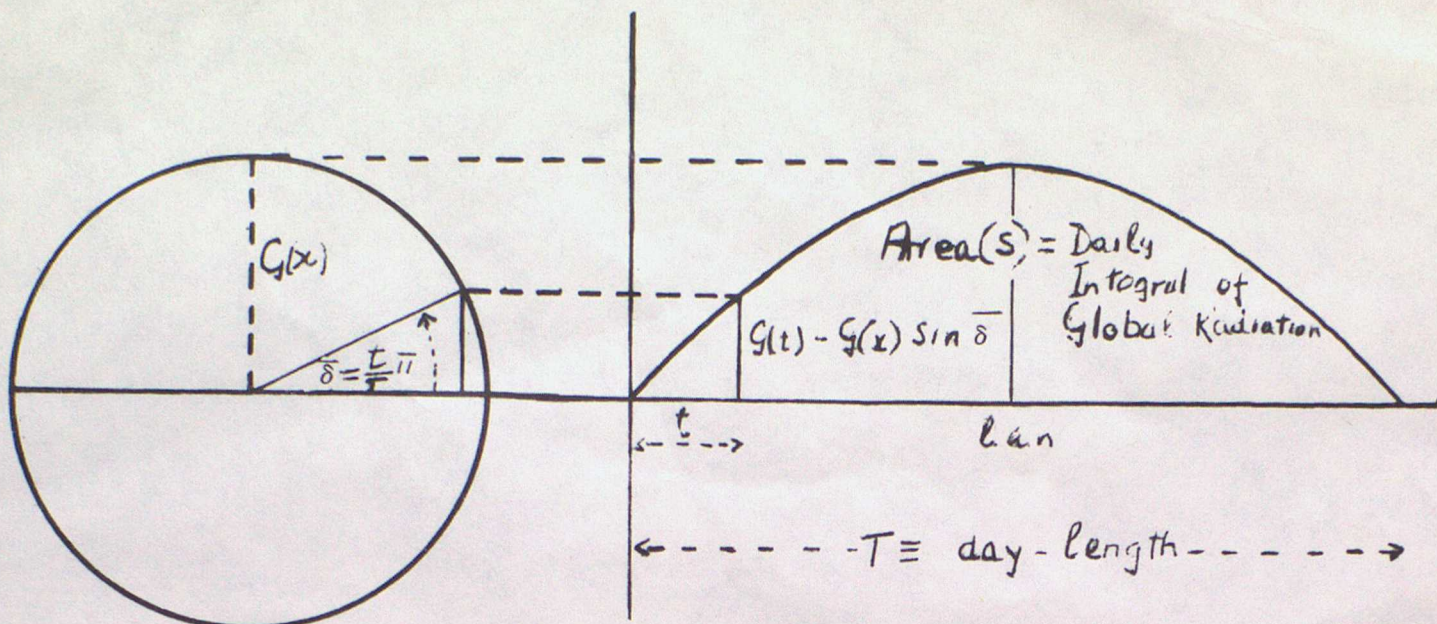
For this particular sample, and month of year, the errors were least when the expedient in (e) was adopted (viz method (2) with a 30 min displacement of the curve). Such a procedure is anyrate consistent with the frequently cited finding that, for a given solar altitude, radiation tends to be more intense before local noon than after local noon.

CONCLUSIONS

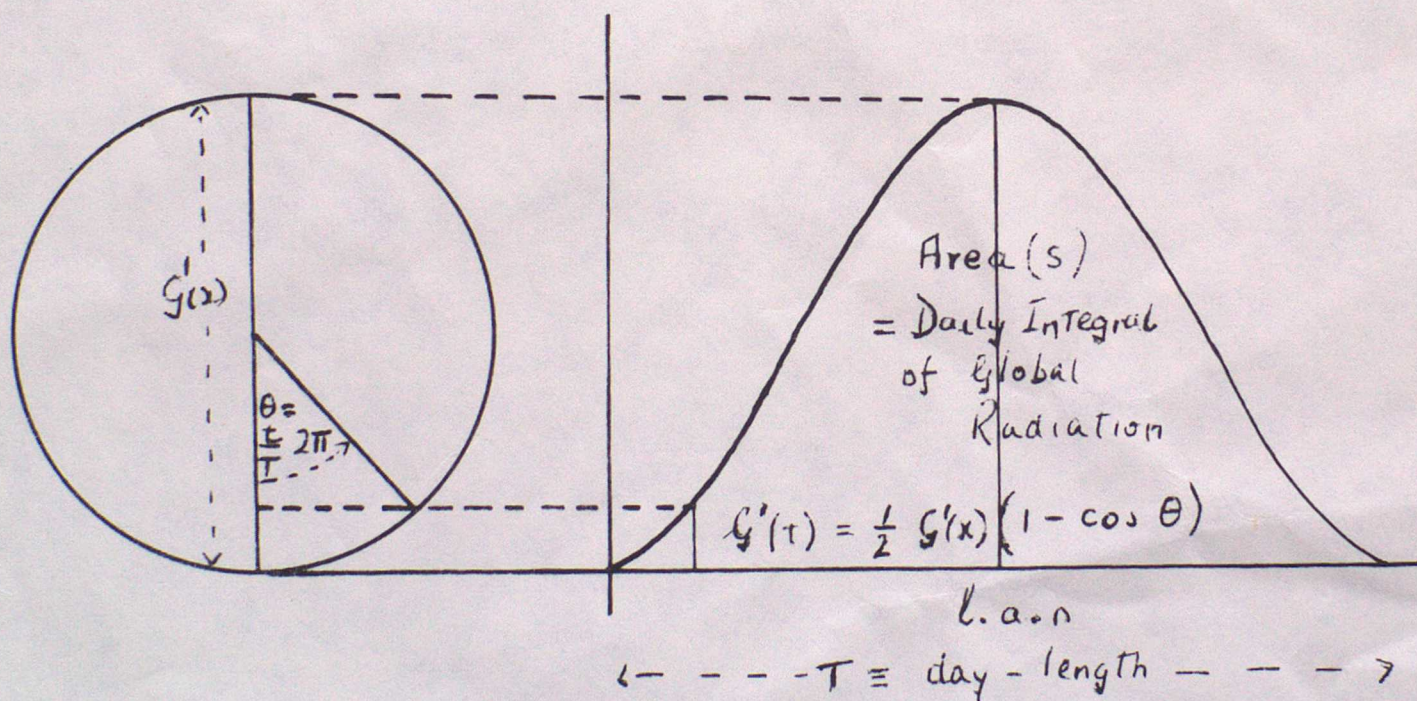
- (i) From a limited sample, it has been found that many features of the diurnal course of mean daily total radiation on a horizontal surface (meaned over at least periods of 10 days) can be reproduced from a knowledge of mean daily total and daylength by either the upper portion of a sine wave (ie $\pm \pi/2$ about an origin at local noon - Monteith) or a complete sine wave (Gloyne).
- (ii) For estimates of the duration above, below or between stated levels of intensity the value obtained by meaning the two estimates appears the most successful. (The error usually being a fraction of an hour).
- (iii) Except perhaps in the winter months, there are indications that the complete sine wave gives a better representation of the daily course - at anyrate in higher latitude regions.
- (iv) Obviously the above findings should be subjected to extensive tests before a firm recommendation on the best procedures is possible.

REFERENCES

- (1) Monteith (1965) "Light distribution and Photosynthesis in Field Crops"
Ann of Bot 29 (New Series) 1965 pp 17-37



(a) After Monteith (1965)



(b) After Gloyne

Fig 1 Co-ordinate Systems for estimation of diurnal course of Global Radiation on a horizontal surface - given

S = mean daily integral

T = length of day

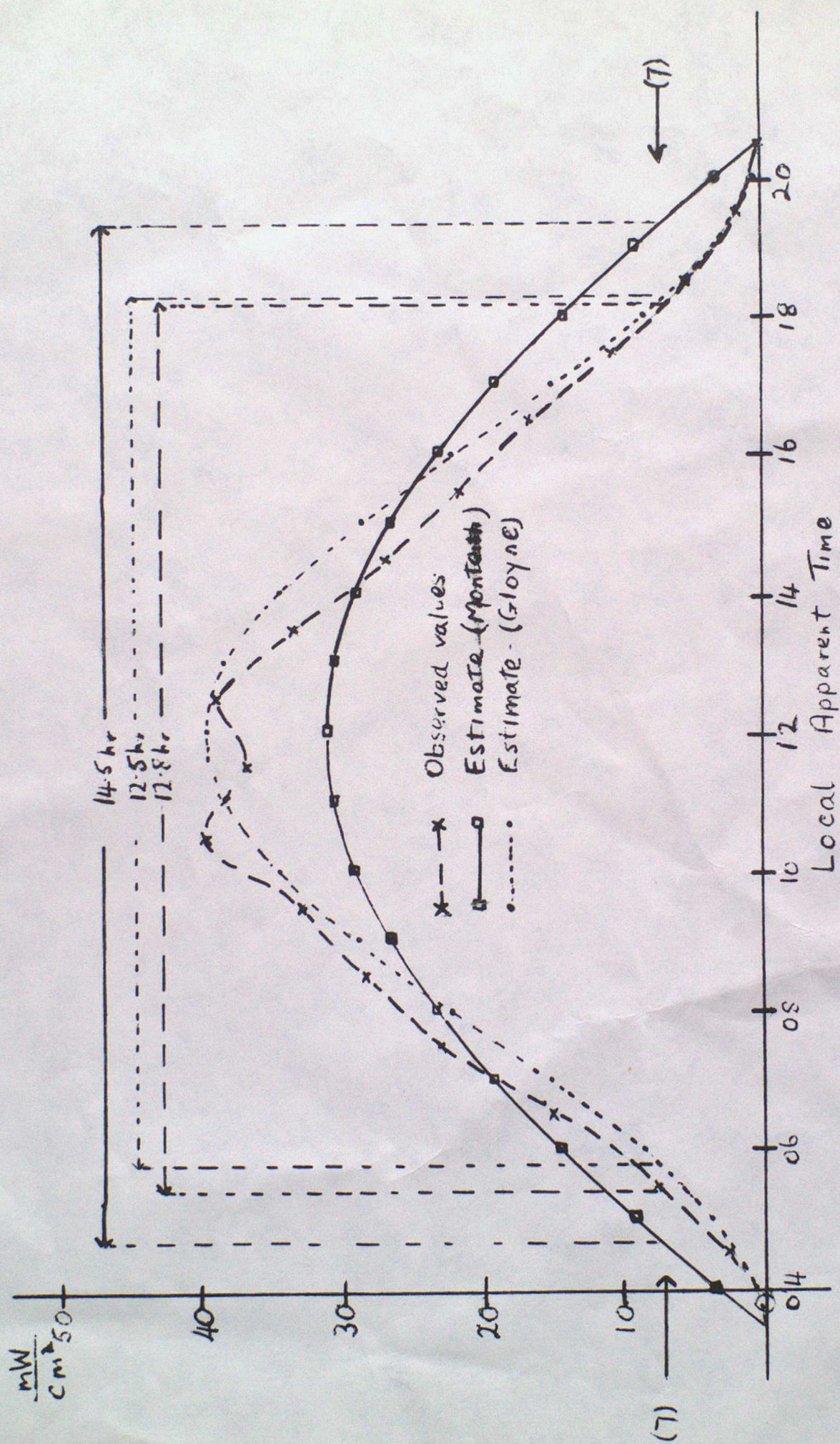


Fig 2 Diurnal course of mean daily global radiation for Aberdeen. July 1970